pcim ASIA

A Model Predictive Method for Specific Harmonic Reduction at Low Switching Frequency in Permanent Magnet Synchronous Motor

Presenter: Tianyu Yuan, North China University of Technology

Coauthors: Xiaoguang Zhang, Yuanhang Cao

Background - PMSM

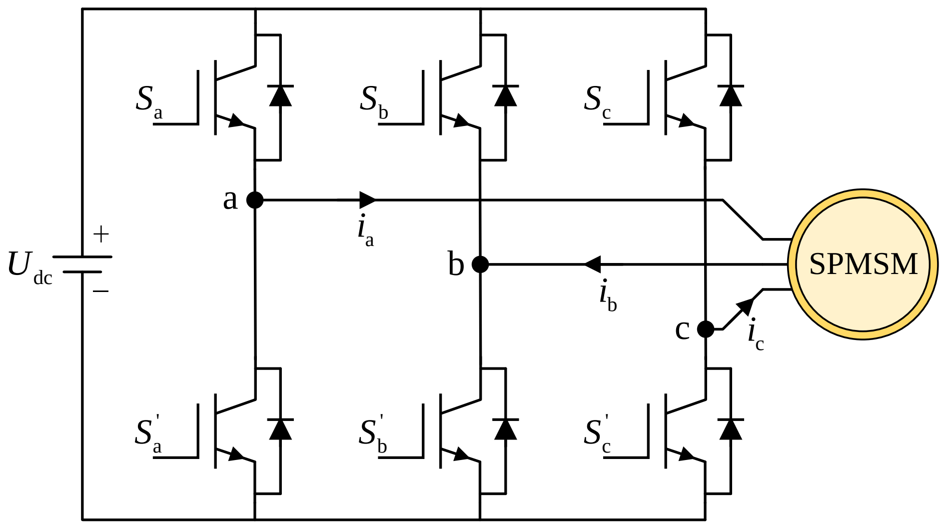


Fig. 1. Main circuit of PMSM drive system.

➢ ***Advantages of PMSM:***

- ✓ Many industrial production applications.
- ✓ Higher power density.
- ✓ Lower operating cost.

Fig. 2. 10,400kW "GuoNeng", PMSM direct drive heavy duty electric locomotive

Fig. 3. B2 Platform Train, Changsha Metro Line 5, using PMSM traction system

Background - MPC

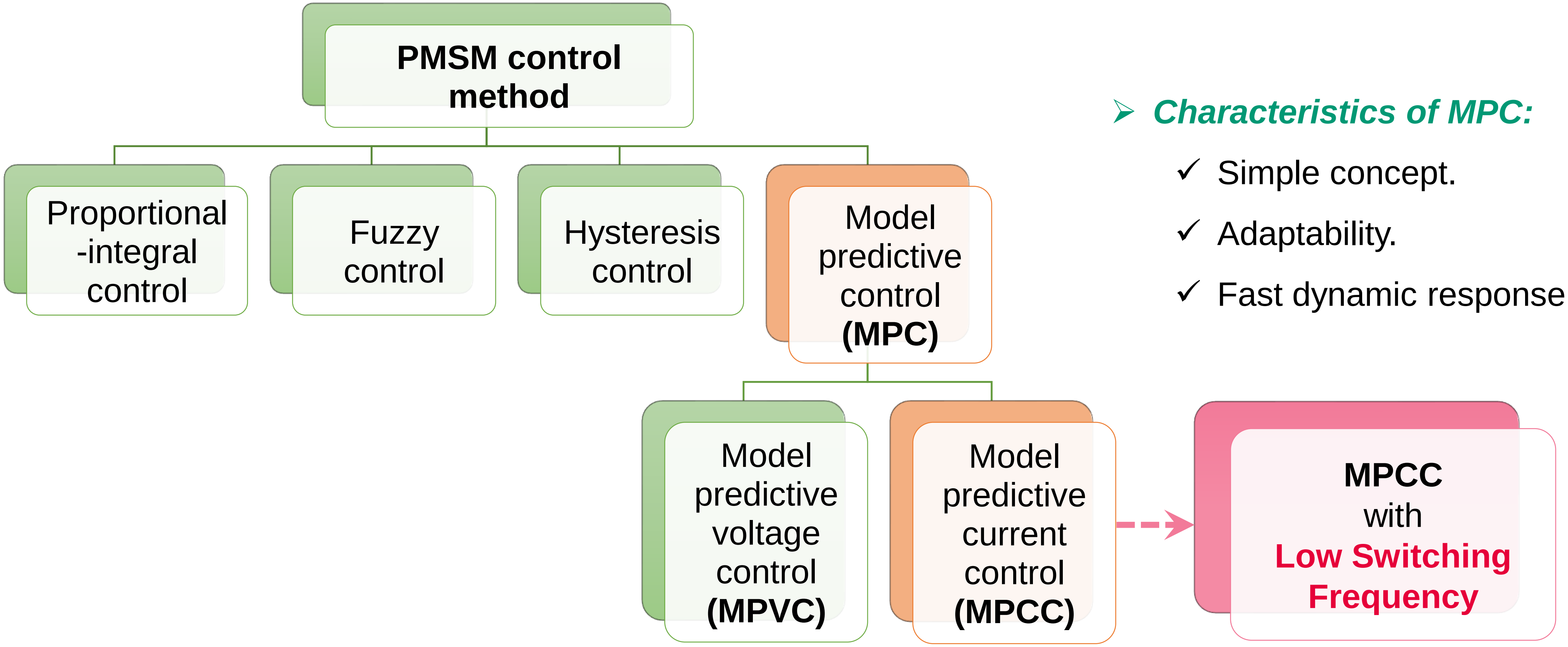


Fig. 2. PMSM control method

- ***Characteristics of MPC:***
  - ✓ Simple concept.
  - ✓ Adaptability.
  - ✓ Fast dynamic response.

T-LSF-MPCC

pcim ASIA

$$\begin{cases} u_d = Ri_d + L\frac{di_d}{dt} - \omega_e L i_q \\ u_q = Ri_q + L\frac{di_q}{dt} + \omega_e L i_d + \omega_e \psi_f \end{cases}$$

Traditional PMSM model

➢ ***Shortcomings in T-LSF-MPCC method:***

- Frequency reduction limit only.
- Makign more lower(5th,7th etc.) harmonics*.
- Harmonic parts are not included in the model.
- Harmonic current prediction is not possible.

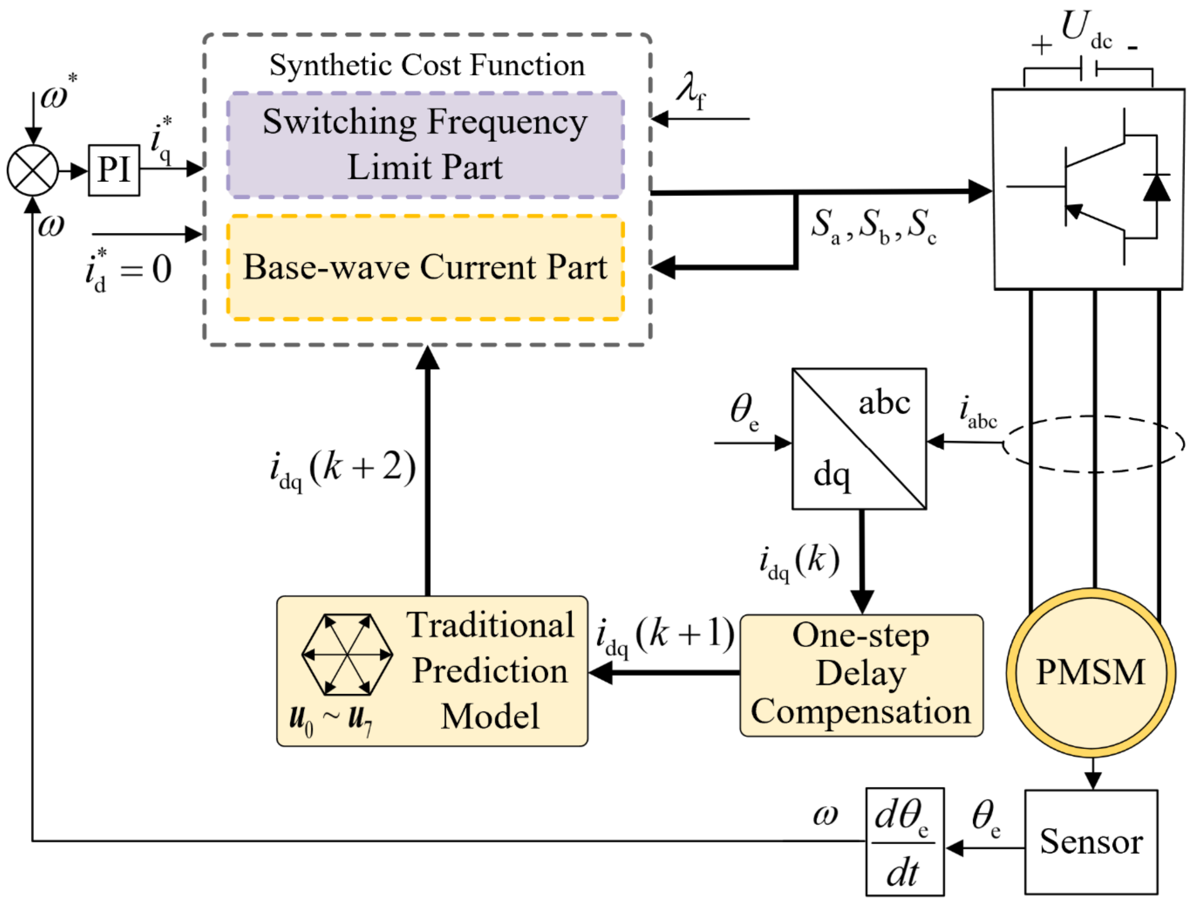


Fig. 4. Block diagram of T-LSF-MPCC Method

***** Z. Chen et.al, "Evaluation of the Harmonics in PMSM with Low Switching Frequency Power Supply"

Method - Objective

pcim ASIA

➢ ***To address the shortcomings of T-LSF-MPCC, the problems that should be solved in this study are as follows:***

Model → **Predictive** → **Control**

- 1. Organize the PMSM model containing harmonic currents.
- 2. Extract harmonic currents.
- 3. Combined model current prediction.
- 4. Integration with "low switching frequency".

Model Predictive Specific Harmonic Reduction Control (MPSHRC)

is proposed

Method - Overview

pcim ASIA

- ***Core parts of MPSHRC method:***

- 1. Harmonic Current Extractor;
- 2. Harmonic Current Prediction;
- 3. Synthetic Cost Function.

Consist

Model → **Predictive** → **Control**

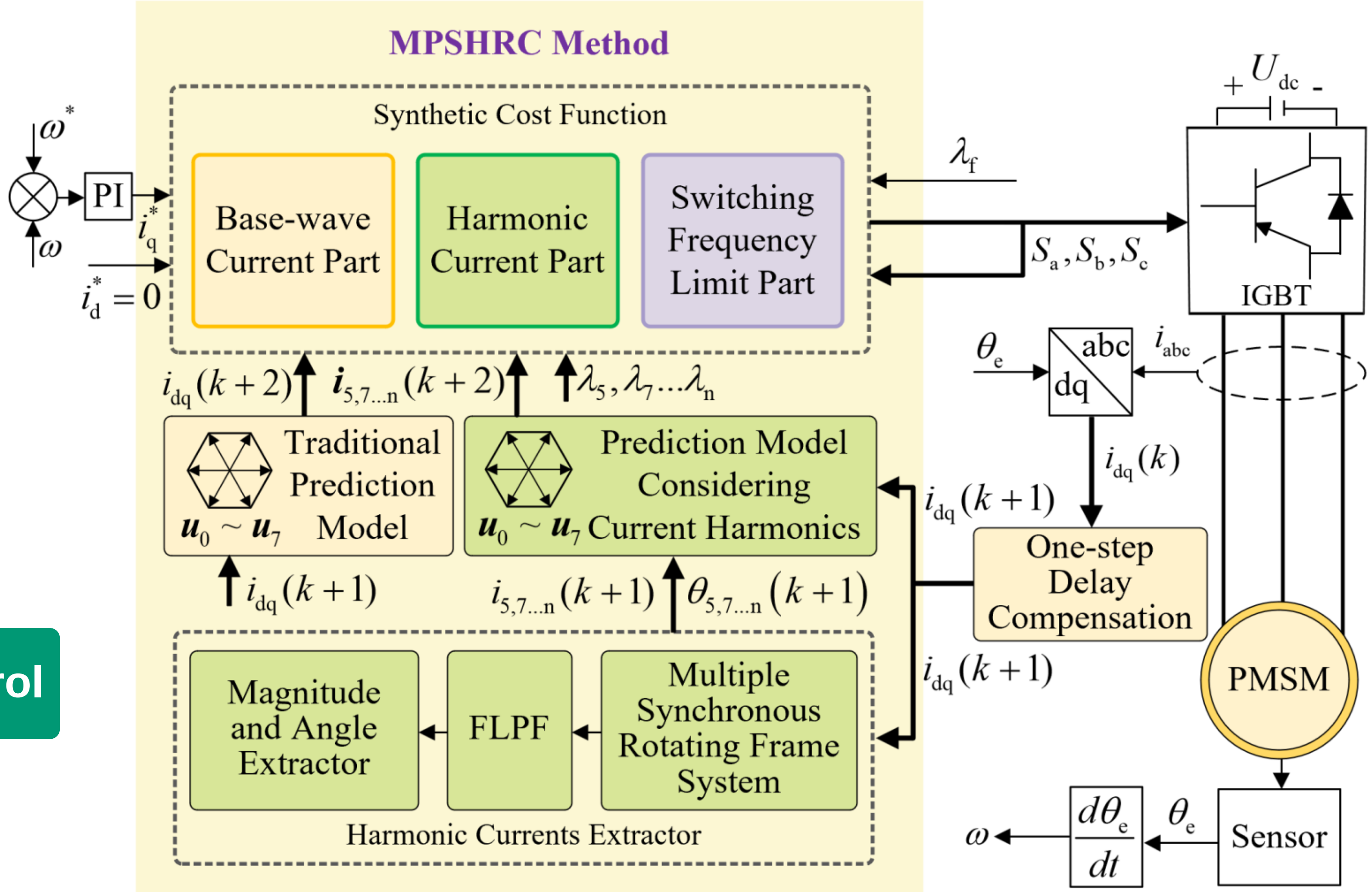


Fig. 5. Block diagram of proposed MPSHRC Method

Method - Elaboration

pcim ASIA

1. Harmonic Current Extractor

- **1.1 PMSM Current Equation Considering Harmonics**

$$\begin{cases} i_{\mathrm{d}} = i_{\mathrm{d}}' + \sum\limits_{n=6k\pm1,k\geq1}^{k\to\infty} \boldsymbol{i}_{\mathrm{d}}^{n} \\ i_{\mathrm{q}} = i_{\mathrm{q}}' + \sum\limits_{n=6k\pm1,k\geq1}^{k\to\infty} \boldsymbol{i}_{\mathrm{q}}^{n} \end{cases} \quad (1)$$

Forms of value and phase angle :

$$\begin{cases} i_{\mathrm{d}} = i_{1}\cos\theta_{1} + \sum\limits_{n=6k\pm1,k\geq1}^{k\to\infty} i_{n}\cos\left(6k\omega_{\mathrm{e}}t + \theta_{n}\right) \\ i_{\mathrm{q}} = i_{1}\sin\theta_{1} + \sum\limits_{n=6k\pm1,k\geq1}^{k\to\infty} i_{n}\sin\left(6k\omega_{\mathrm{e}}t + \theta_{n}\right) \end{cases} \quad (2)$$

Rotation speed

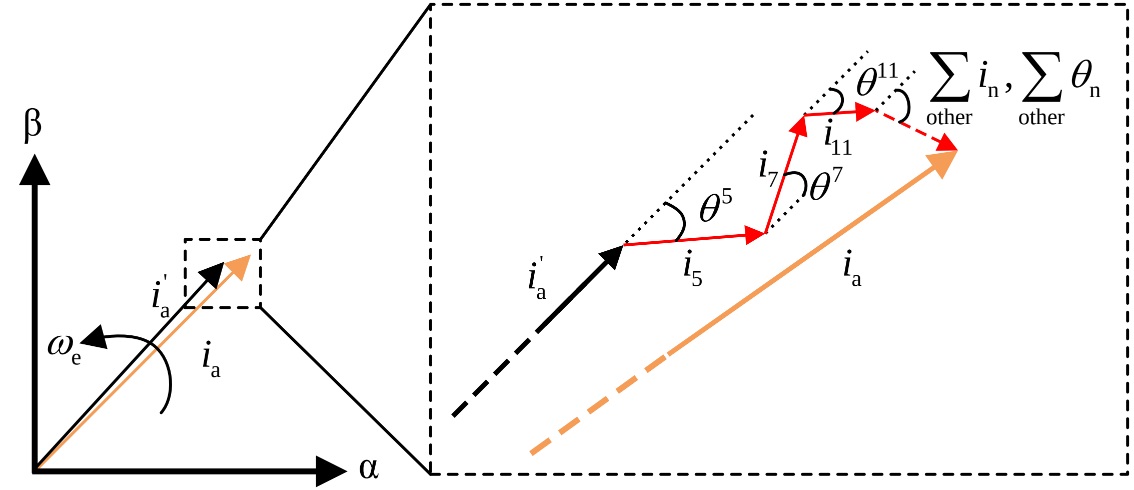


Fig. 6. PMSM stator current with all 6k±1 current harmonics in the αβ axes.

Method - Elaboration

pcim ASIA

1. Harmonic Current Extractor

➢ **1.1 PMSM Current Equation Considering Harmonics**

$$\begin{cases} i_{\mathrm{d}} = i_{\mathrm{d}}^{'} + i_{\mathrm{d}}^{5} + i_{\mathrm{d}}^{7} = i_1 \cos\theta_1 + i_5 \cos(6\omega_{\mathrm{e}} t + \theta_5) + i_7 \cos(6\omega_{\mathrm{e}} t + \theta_7) \\ i_{\mathrm{q}} = i_{\mathrm{q}}^{'} + i_{\mathrm{q}}^{5} + i_{\mathrm{q}}^{7} = i_1 \sin\theta_1 + i_5 \sin(6\omega_{\mathrm{e}} t + \theta_5) + i_7 \sin(6\omega_{\mathrm{e}} t + \theta_7) \end{cases} \quad (3)$$

substitute into

$$\begin{cases} u_{\mathrm{d}} = Ri_{\mathrm{d}} + L\dfrac{\mathrm{d}i_{\mathrm{d}}}{\mathrm{d}t} - \omega_{\mathrm{e}} L i_{\mathrm{q}} \\ u_{\mathrm{q}} = Ri_{\mathrm{q}} + L\dfrac{\mathrm{d}i_{\mathrm{q}}}{\mathrm{d}t} + \omega_{\mathrm{e}} L i_{\mathrm{d}} + \omega_{\mathrm{e}} \psi_{\mathrm{f}} \end{cases} \quad (4)$$

get

$$\begin{cases} u_{\mathrm{d}} = R(i_{\mathrm{d}}^{'} + i_{\mathrm{d}}^{5} + i_{\mathrm{d}}^{7}) + L\left(\dfrac{\mathrm{d}}{\mathrm{d}t}(i_{\mathrm{d}}^{'} + i_{\mathrm{d}}^{5} + i_{\mathrm{d}}^{7}) - \omega_{\mathrm{e}}(i_{\mathrm{q}}^{'} + i_{\mathrm{q}}^{5} + i_{\mathrm{q}}^{7})\right) \\ u_{\mathrm{q}} = R(i_{\mathrm{q}}^{'} + i_{\mathrm{q}}^{5} + i_{\mathrm{q}}^{7}) + +L\left(\dfrac{\mathrm{d}}{\mathrm{d}t}(i_{\mathrm{q}}^{'} + i_{\mathrm{q}}^{5} + i_{\mathrm{q}}^{7}) + \omega_{\mathrm{e}}(i_{\mathrm{d}}^{'} + i_{\mathrm{d}}^{5} + i_{\mathrm{d}}^{7})\right) + \omega_{\mathrm{e}} \psi_{\mathrm{f}} \end{cases} \quad (5)$$

Voltage equation for dq-axes considering harmonic currents, Will be used in the method analysis

Method - Elaboration

pcim ASIA

1. Harmonic Current Extractor

➢ **1.2 Multiple Synchronous Rotating Frame System**

$$\begin{bmatrix} i_d^{5F} \\ i_q^{5F} \\ i_d^{7F} \\ i_q^{7F} \end{bmatrix} = \begin{bmatrix} \cos(5\omega_e t) & -\sin(5\omega_e t) \\ \sin(5\omega_e t) & \cos(5\omega_e t) \\ \cos(7\omega_e t) & \sin(7\omega_e t) \\ -\sin(7\omega_e t) & \cos(7\omega_e t) \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (6)$$

Note: Only the fundamental frequency axes and the 5th and 7th order axes are listed.

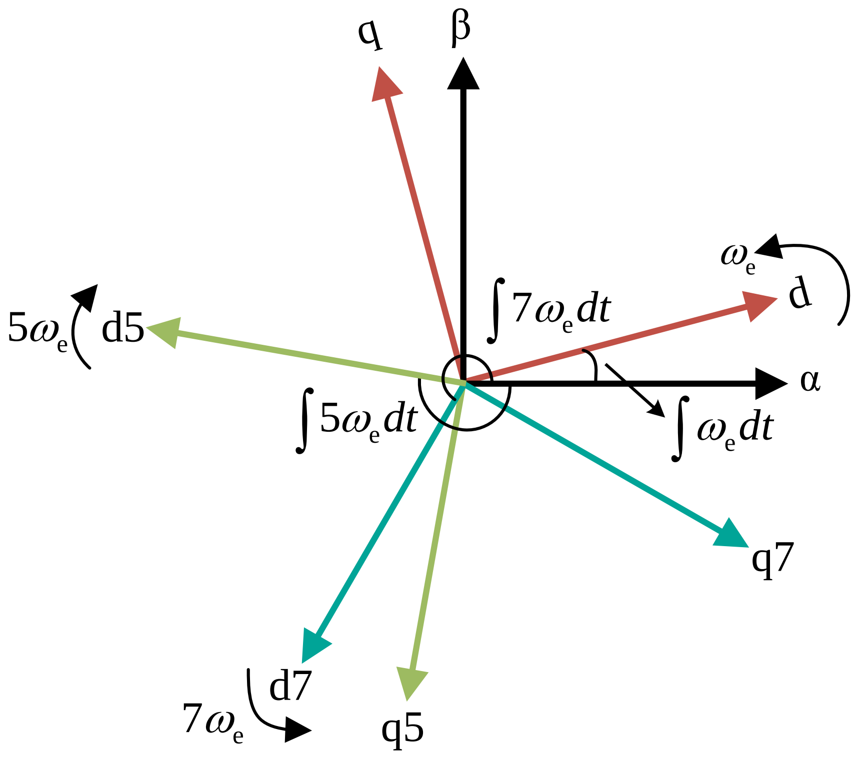


Fig. 7. MSRFS Method.

Method - Elaboration

pcim ASIA

1. Harmonic Current Extractor

➢ **1.3 Calculation of Harmonic Currents in Each dq axes**

Harmonic current extraction using first-order low-pass filter(FLPF). The input-output relation equation of the FLPF after back-ward Eulerian discretization is

$$y(k) = \frac{2\pi f_n T_s}{1 + 2\pi f_n T_s} x(k) + \frac{1}{1 + 2\pi f_n T_s} y(k-1) \quad (7)$$

The cutoff frequency is set to run at **10 Hz**, which is much lower than the 6th harmonic frequency in synchronous rotating coordinate system.

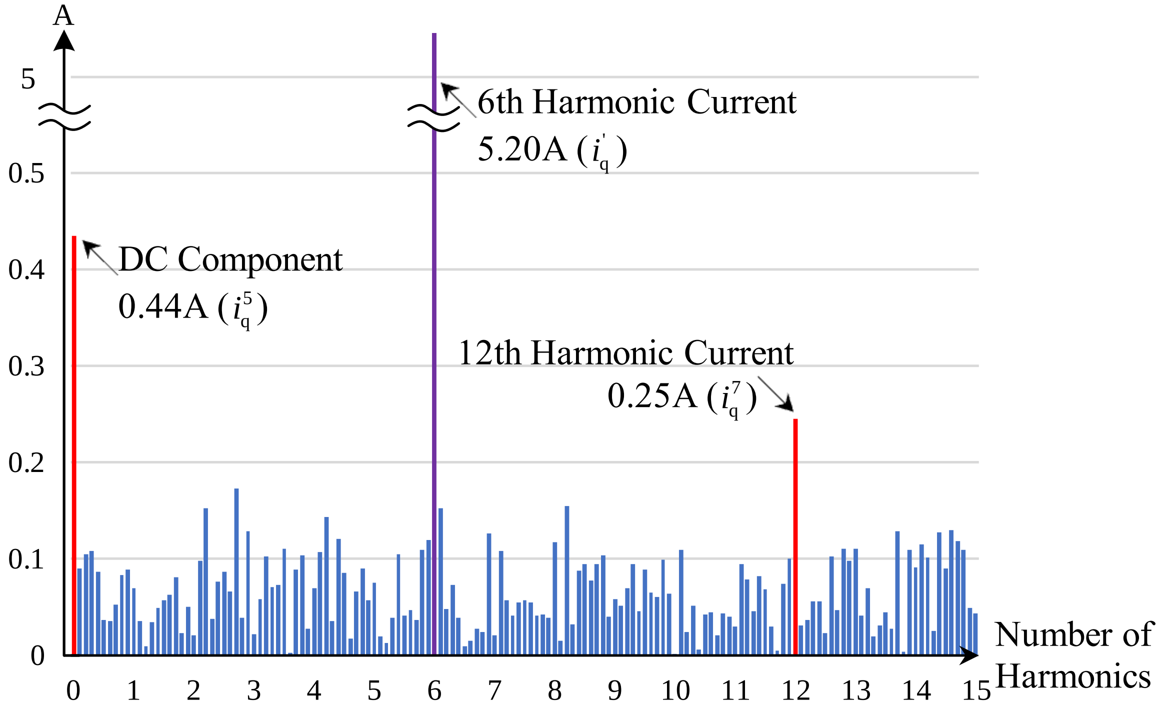


Fig. 8. FFT analysis of i_q^{5F}.

Method - Elaboration

pcim ASIA

1. Harmonic Current Extractor

➢ **1.3 Calculation of Harmonic Currents in Each dq axes**

Vectors synthesized from 5th,7th dq axes harmonic currents can be calculated by

$$\begin{cases} i_5 = \sqrt{\left(i_d^5\right)^2 + \left(i_q^5\right)^2}, i_7 = \sqrt{\left(i_d^7\right)^2 + \left(i_q^7\right)^2} \\ \theta_5 = \arctan\frac{i_q^5}{i_d^5}, \theta_7 = \arctan\frac{i_q^7}{i_d^7} \\ \boldsymbol{i}_5 = i_5 e^{j\theta_5}, \quad \boldsymbol{i}_7 = i_7 e^{j\theta_7} \end{cases} \tag{8}$$

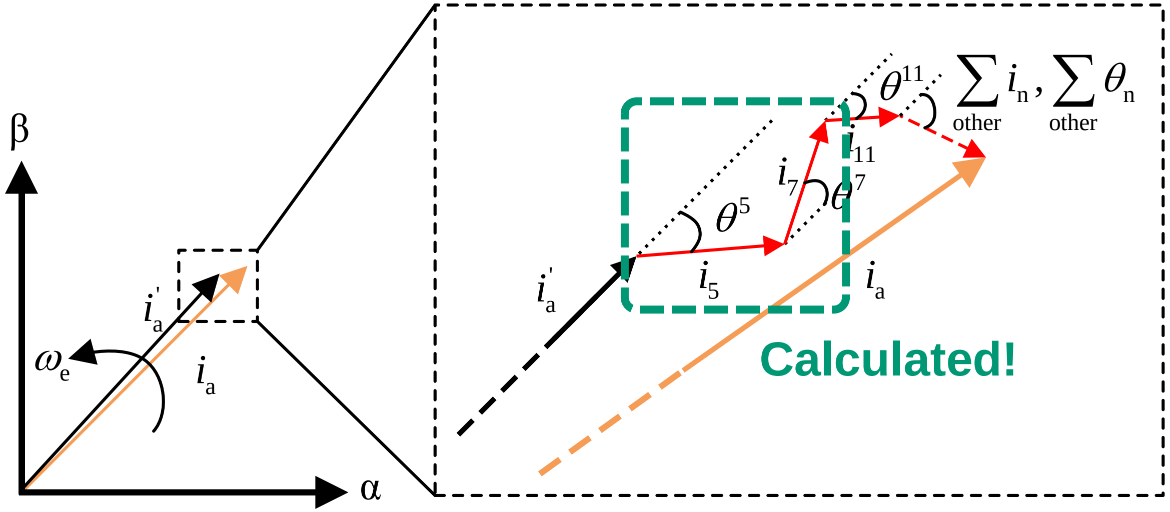


Fig. 6. PMSM stator current with all 6k±1 current harmonics in the αβ axes.

Next step: current prediction

Method - Elaboration

pcim ASIA

2. Harmonic Current Prediction

➢ **2.1 Prediction Model Considering Current Harmonics**

Starting with traditional model...

$$\begin{cases} u_{\mathrm{d}} = Ri_{\mathrm{d}} + L\dfrac{\mathrm{d}i_{\mathrm{d}}}{\mathrm{d}t} - \omega_{\mathrm{e}} L i_{\mathrm{q}} \\ u_{\mathrm{q}} = Ri_{\mathrm{q}} + L\dfrac{\mathrm{d}i_{\mathrm{q}}}{\mathrm{d}t} + \omega_{\mathrm{e}} L i_{\mathrm{d}} + \omega_{\mathrm{e}}\psi_{\mathrm{f}} \end{cases} \quad (9)$$

Euler forward

One-step delay

$$\begin{cases} i_{\mathrm{d}}^{\mathrm{p}}(k+1) = (1-\dfrac{T_{\mathrm{s}}R}{L})i_{\mathrm{d}}(k) + \dfrac{T_{\mathrm{s}}}{L}u_{\mathrm{d}}(k) + T_{\mathrm{s}}\omega_{\mathrm{e}} i_{\mathrm{q}}(k) \\ i_{\mathrm{q}}^{\mathrm{p}}(k+1) = (1-\dfrac{T_{\mathrm{s}}R}{L})i_{\mathrm{q}}(k) + \dfrac{T_{\mathrm{s}}}{L}u_{\mathrm{q}}(k) - T_{\mathrm{s}}\omega_{\mathrm{e}} i_{\mathrm{d}}(k) - \dfrac{T_{\mathrm{s}}\omega_{\mathrm{e}}\psi_{\mathrm{f}}}{L} \end{cases} \quad (10)$$

Euler forward

Prediction

$$\begin{cases} i_{\mathrm{d}}^{\mathrm{p}}(k+2) = (1-\dfrac{T_{\mathrm{s}}R}{L})i_{\mathrm{d}}(k+1) + \dfrac{T_{\mathrm{s}}}{L}u_{\mathrm{d}}(k) + T_{\mathrm{s}}\omega_{\mathrm{e}} i_{\mathrm{q}}(k+1) \\ i_{\mathrm{q}}^{\mathrm{p}}(k+2) = (1-\dfrac{T_{\mathrm{s}}R}{L})i_{\mathrm{q}}(k+1) + \dfrac{T_{\mathrm{s}}}{L}u_{\mathrm{q}}(k) - T_{\mathrm{s}}\omega_{\mathrm{e}} i_{\mathrm{d}}(k+1) - \dfrac{T_{\mathrm{s}}\omega_{\mathrm{e}}\psi_{\mathrm{f}}}{L} \end{cases} \quad (11)$$

Substitute into

$$\begin{cases} i_{\mathrm{d}} = i_{\mathrm{d}}^{'} + i_{\mathrm{d}}^{5} + i_{\mathrm{d}}^{7} \\ i_{\mathrm{q}} = i_{\mathrm{q}}^{'} + i_{\mathrm{q}}^{5} + i_{\mathrm{q}}^{7} \end{cases} \quad (3)$$

Proposed harmonic current equation

Method - Elaboration

pcim ASIA

2. Harmonic Current Prediction

➢ **2.1 Prediction Model Considering Current Harmonics**

$$\begin{cases} i_{d}^{5p}(k+2)=i_{d}^{5p}(k+1)-T_{s}\left(\dfrac{d\left(i_{d}^{'p}+i_{d}^{7p}\right)}{dt}\right)+T_{s}\left(\dfrac{u_{d}-Ri_{d}^{p}(k+1)+\omega_{e}Li_{q}^{p}(k+1)}{L}\right) \\ i_{q}^{5p}(k+2)=i_{q}^{5p}(k+1)-T_{s}\left(\dfrac{d\left(i_{q}^{'p}+i_{q}^{7p}\right)}{dt}\right)+T_{s}\left(\dfrac{u_{d}-Ri_{q}^{p}(k+1)-\omega_{e}\left(Li_{d}^{p}(k+1)+\psi_{f}\right)}{L}\right) \\ i_{d}^{7p}(k+2)=i_{d}^{7p}(k+1)-T_{s}\left(\dfrac{d\left(i_{d}^{'p}+i_{d}^{5p}\right)}{dt}\right)+T_{s}\left(\dfrac{u_{d}-Ri_{d}^{p}(k+1)+\omega_{e}Li_{q}^{p}(k+1)}{L}\right) \\ i_{q}^{7p}(k+2)=i_{q}^{7p}(k+1)-T_{s}\left(\dfrac{d\left(i_{q}^{'p}+i_{q}^{5p}\right)}{dt}\right)+T_{s}\left(\dfrac{u_{d}-Ri_{q}^{p}(k+1)-\omega_{e}\left(Li_{d}^{p}(k+1)+\psi_{f}\right)}{L}\right) \end{cases} \tag{12}$$

Need to define the derivative term

Method - Elaboration

2. Harmonic Current Prediction

➢ **2.1 Prediction Model Considering Current Harmonics**

The derivative of each harmonic is defined as the value of the harmonic current derivative at the previous moment, which can be expressed as

$$\begin{cases} \dfrac{di_{\mathrm{d}}^{'\mathrm{p}}}{dt} = \dfrac{i_{\mathrm{d}}^{'\mathrm{p}}(k+1) - i_{\mathrm{d}}^{'}(k)}{T_{\mathrm{s}}}, \dfrac{di_{\mathrm{q}}^{'\mathrm{p}}}{dt} = \dfrac{i_{\mathrm{q}}^{'\mathrm{p}}(k+1) - i_{\mathrm{q}}^{'}(k)}{T_{\mathrm{s}}} \\ \dfrac{di_{\mathrm{d}}^{5\mathrm{p}}}{dt} = \dfrac{i_{\mathrm{d}}^{5\mathrm{p}}(k+1) - i_{\mathrm{d}}^{5}(k)}{T_{\mathrm{s}}}, \dfrac{di_{\mathrm{q}}^{5\mathrm{p}}}{dt} = \dfrac{i_{\mathrm{q}}^{5\mathrm{p}}(k+1) - i_{\mathrm{q}}^{5}(k)}{T_{\mathrm{s}}} \\ \dfrac{di_{\mathrm{d}}^{7\mathrm{p}}}{dt} = \dfrac{i_{\mathrm{d}}^{7\mathrm{p}}(k+1) - i_{\mathrm{d}}^{7}(k)}{T_{\mathrm{s}}}, \dfrac{di_{\mathrm{q}}^{7\mathrm{p}}}{dt} = \dfrac{i_{\mathrm{q}}^{7\mathrm{p}}(k+1) - i_{\mathrm{q}}^{7}(k)}{T_{\mathrm{s}}} \end{cases} \quad (13)$$

It should be noted that the above equations is not limited to the 5th ,7th harmonics and can be further expanded.

Method - Elaboration

pcim ASIA

2. Harmonic Current Prediction

➢ **2.2 Traditional Prediction Model**

$$\begin{cases} i_d^p(k+2) = (1-\frac{T_s R}{L})i_d(k+1) + \frac{T_s}{L}u_d(k) + T_s\omega_e i_q(k+1) \\ i_q^p(k+2) = (1-\frac{T_s R}{L})i_q(k+1) + \frac{T_s}{L}u_q(k) - T_s\omega_e i_d(k+1) - \frac{T_s\omega_e\psi_f}{L} \end{cases}$$

*Reasons for using **both models** together?*

- Given quantities $\boldsymbol{i}_{dq}^*$ have dynamic characteristic;
- If only a harmonic model is used, the given quantities also need to be extracted harmonically;
- Will result in reduced dynamic effect and increased computational effort.

Next step: Synthetic Cost Function

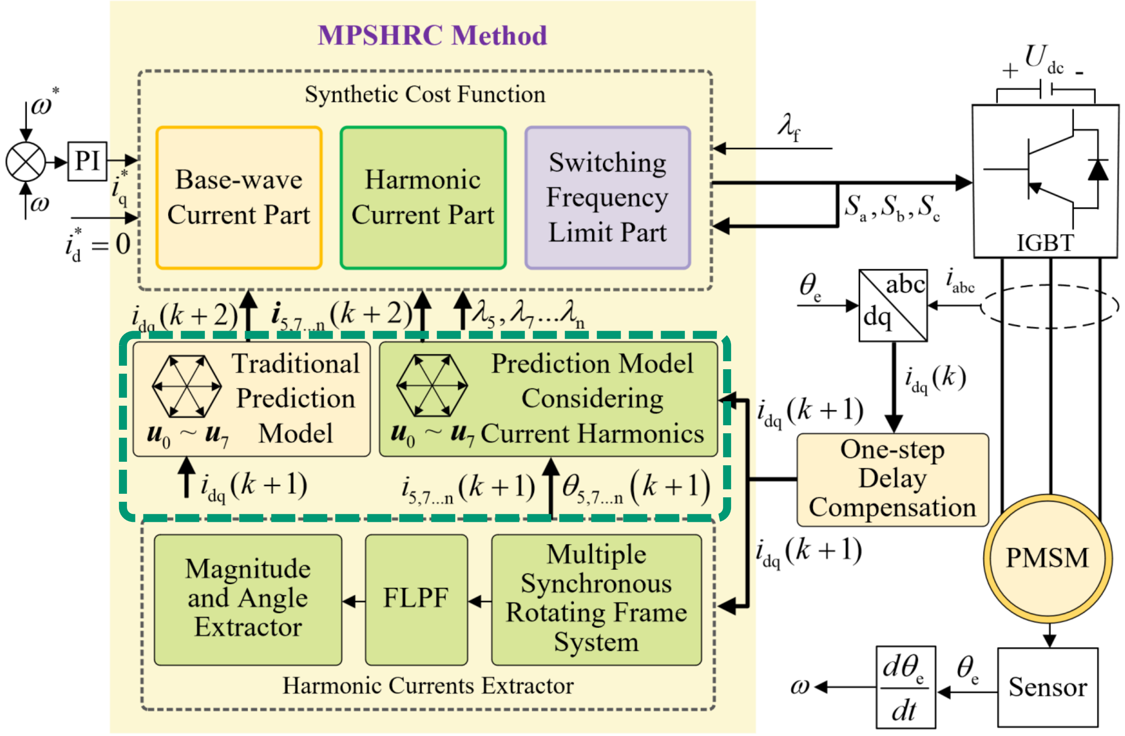


Fig. 5. Block diagram of proposed MPSHRC Method

Method - Elaboration

pcim ASIA

3. Synthetic Cost Function

$$
\begin{cases}
J = g_{\mathrm{b}} + g_{\mathrm{h}} + g_{\mathrm{s}} \\
g_{\mathrm{b}} = \left[i_{\mathrm{d}}^{*} - i_{\mathrm{d}}^{\mathrm{p}}(k+2) \right]^2 + \left[i_{\mathrm{q}}^{*} - i_{\mathrm{q}}^{\mathrm{p}}(k+2) \right]^2 \\
g_{\mathrm{h}} = \lambda_5 \left(\left(i_{\mathrm{d}}^{5}(k+1) \right)^2 + \left(i_{\mathrm{d}}^{5}(k+1) \right)^2 \right) \\
\qquad + \lambda_7 \left(\left(i_{\mathrm{d}}^{7}(k+1) \right)^2 + \left(i_{\mathrm{d}}^{7}(k+1) \right)^2 \right) \\
g_{\mathrm{s}} = \lambda_{\mathrm{f}} \sum_{i=a,b,c} \left| S_i(k) - S_i(k-1) \right|
\end{cases}
\tag{14}
$$

Selection of weighting factors:

- Calculation and analysis first;
- Further changing based on real results.

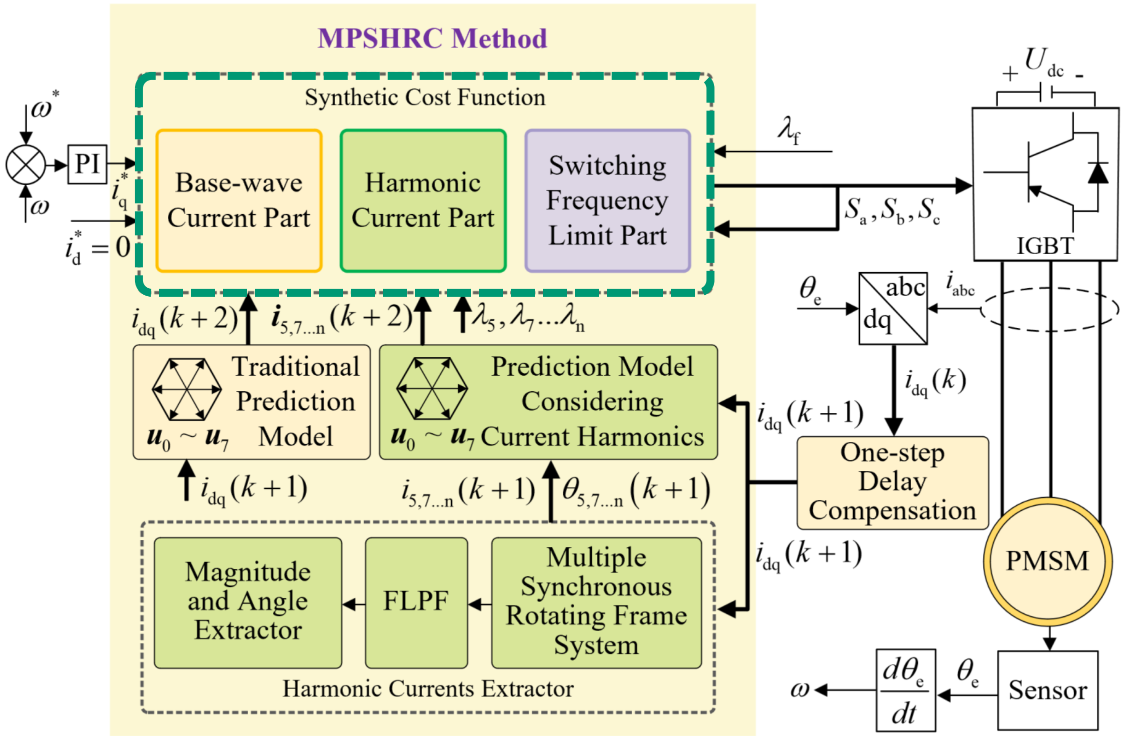


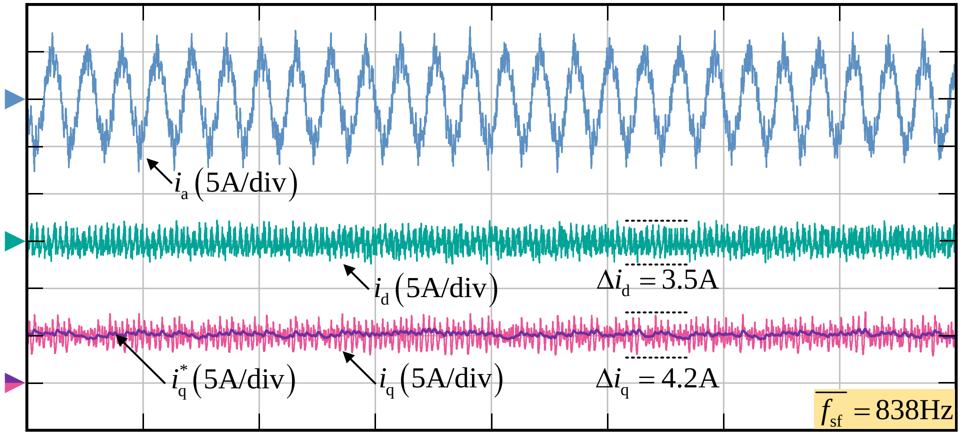
Fig. 5. Block diagram of proposed MPSHRC Method

Results – *High* speed

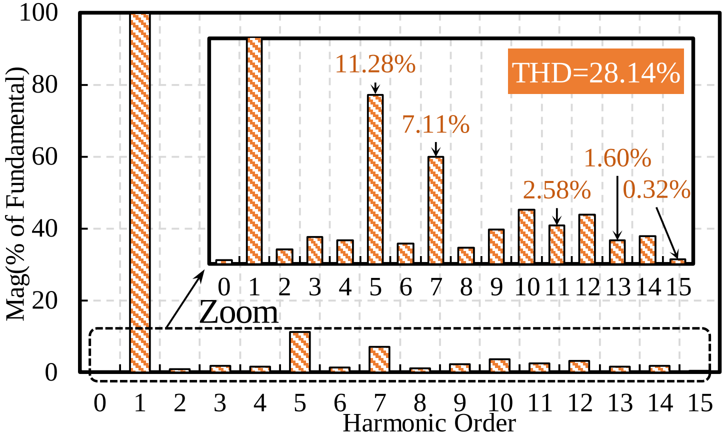
pcim ASIA

The average switching frequency $\overline{f_L}$ is **830Hz**.

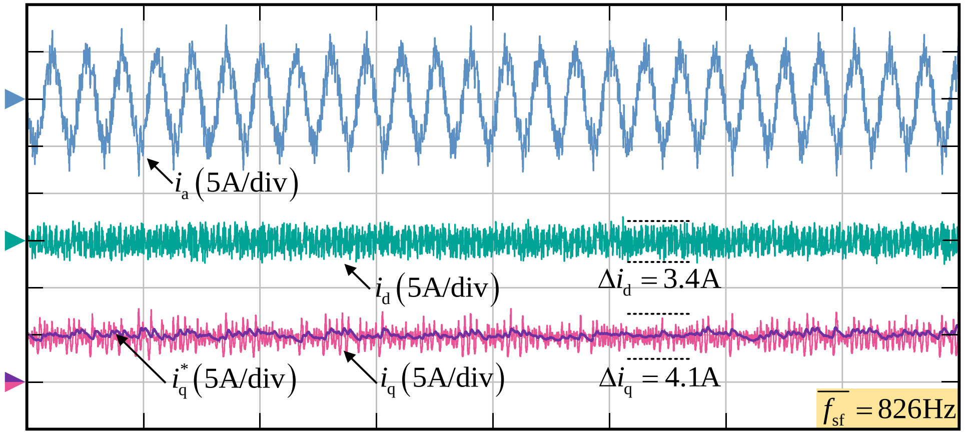
T-LSF-MPCC method



THD Analysis of stator current



Proposed MPSHRC method



THD Analysis of stator current

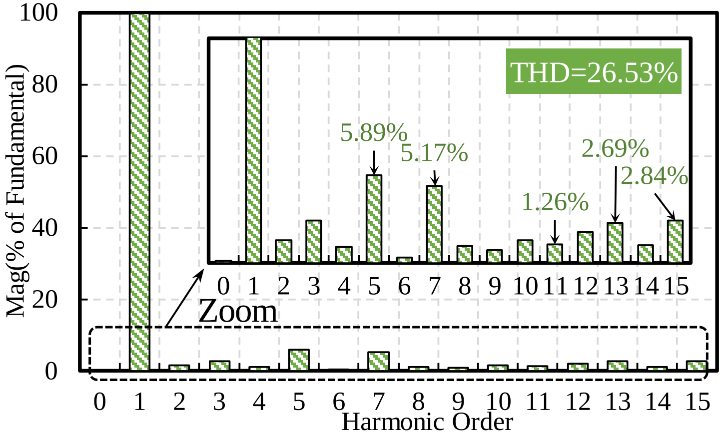


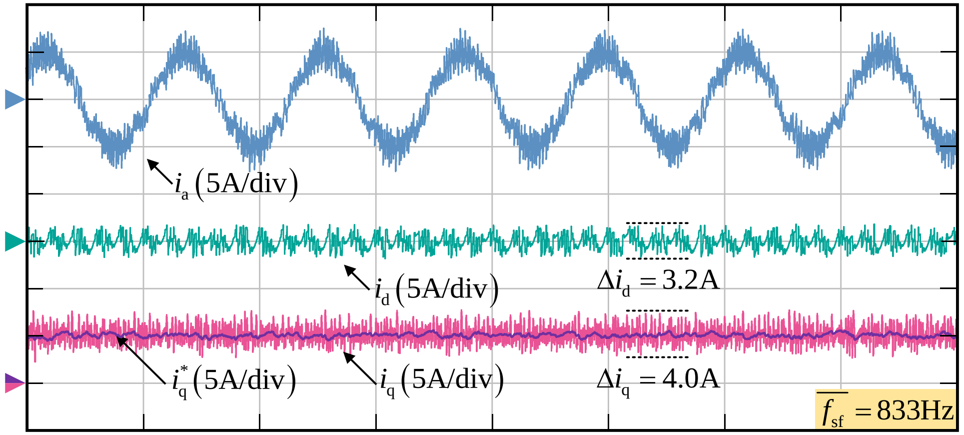
Fig. 9. Simulation results at rated torque (5N.m) and **rated speed(2000rad/min)**

Results – *Low* speed

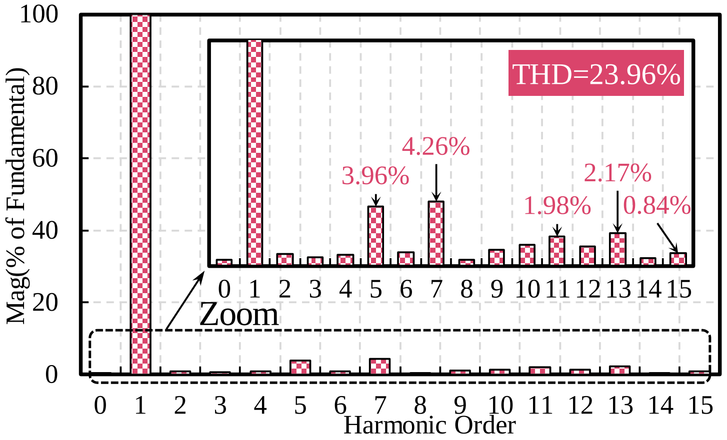
pcim ASIA

The average switching frequency $\overline{f_L}$ is **830Hz**.

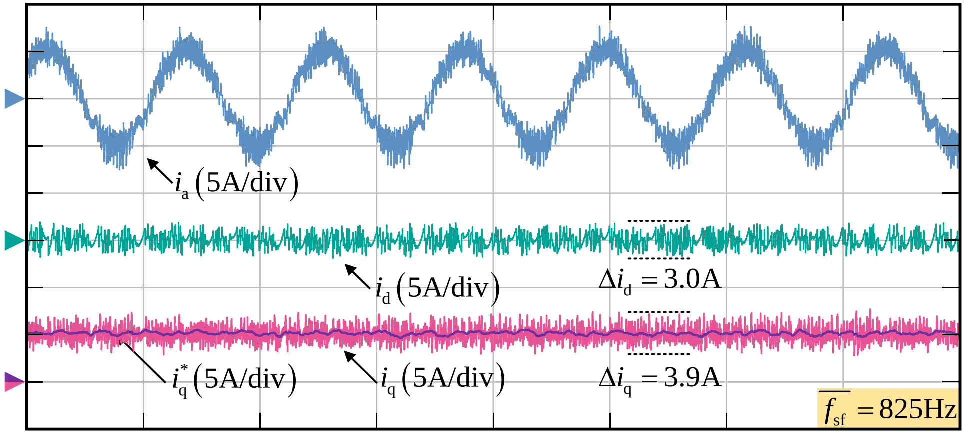
T-LSF-MPCC method



THD Analysis of stator current



Proposed MPSHRC method



THD Analysis of stator current

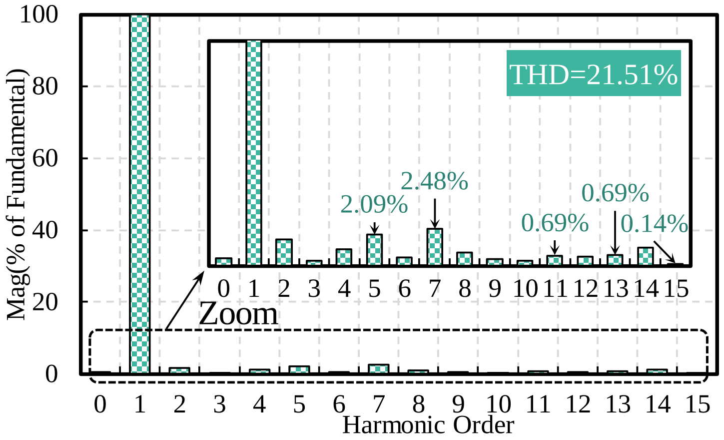


Fig. 10. Simulation results at rated torque (5N.m) and **low speed(500rad/min)**

Conclusion

pcim ASIA

- Considering the impact of low harmonics on PMSMs operating at low switching frequencies, This paper proposes a specific number of harmonics reduction method based on model predictive control.
- The method extracts the corresponding harmonics using **MSRFS** and uses the **next moment harmonic quantities predicted** by a mathematical model of the PMSM considering the harmonics, and **a synthetic cost function** is designed to optimize the system in conjunction with a finite-set model predictive control.
- Both demonstration and simulation validate the effectiveness of the proposed methodology and detailed data indications are given.

Thanks for your listening!

A Model Predictive Method for Specific Harmonic Reduction at Low Switching Frequency in Permanent Magnet Synchronous Motor

Presenter: Tianyu Yuan, North China University of Technology
Coauthors: Xiaoguang Zhang, Yuanhang Cao