

# Parameter Estimation of DTP PMSM Based on the Recursive Least Square Method

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1. Research Background

2. Mathematical Model of DTP-PMSM

3. DTP-PMSM Parameter Estimation strategy

Base on Recursive Least Square Method

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# Research Background

Dual three phase permanent magnet synchronous motor(DTP PMSM) possess fault-tolerant capability and have found extensive application in electric propulsion system.



Electric Propulsion Aircraft



Electric Vehicle

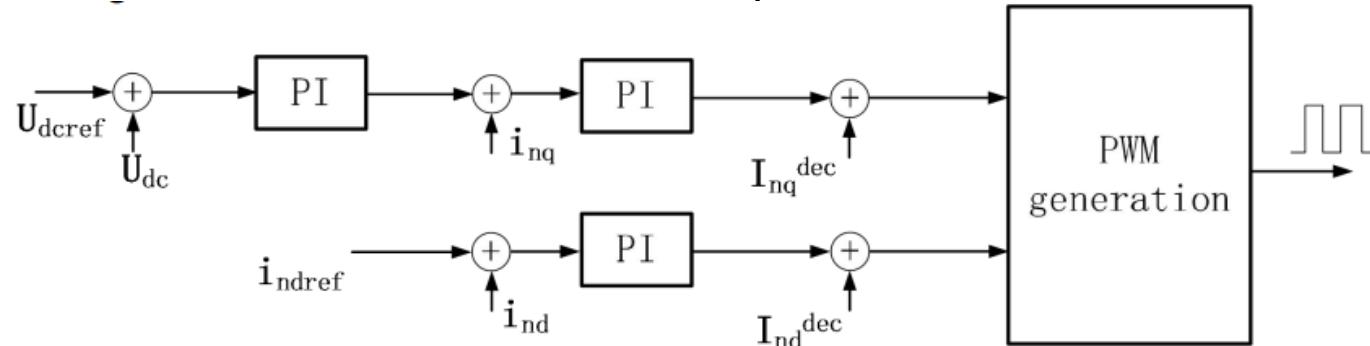


Rail Traffic

During the operation of a DTP PMSM, variations in parameters due to magnetic field saturation and temperature can impact the dynamic response and control accuracy, and the performance of associated algorithms, including those relying on motor parameters for sensorless control.

# FOC—Feedforward Decoupling Link

In order to improve the system control dynamic performance, advanced feed coupling compensation needs to be added to the current closed loop



The stator voltage equation and the flux equation:

$$u_{ns} = R_n i_{ns} + \frac{d\psi_{ns}}{dt}$$

$$\psi_{ns} = L_{ns} i_{ns} + \lambda_{ns} \psi_f$$

Coordinate transformation matrix

$$T_{ns/dq} \cdot u_{ns} = T_{ns/dq} \cdot R_n i_{ns} + T_{ns/dq} \cdot \frac{d\psi_{ns}}{dt}$$

$$T_{ns/dq} \cdot \frac{d\psi_{ns}}{dt} = \frac{dT_{ns/dq} \cdot \psi_{ns}}{dt} - \psi_{ns} \cdot \frac{dT_{ns/dq}}{dt}$$

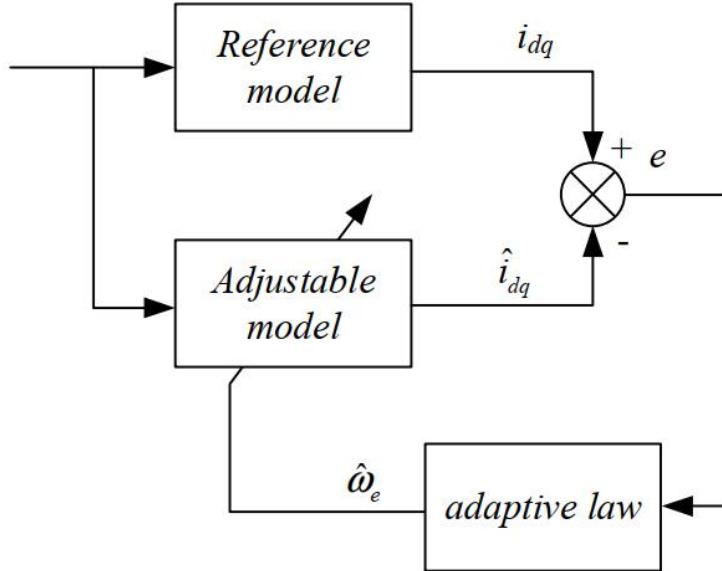
Feedforward decoupling voltage common component:

$$u_{ndq} = T_{ns/dq} \cdot R_n \cdot T_{ns/dq}^{-1} i_{dq} + T_{ns/dq} \cdot L_{ns} \cdot T_{ns/dq}^{-1} \frac{di_{dq}}{dt} - \frac{dT_{ns/dq}}{dt} \cdot L_{ns} \cdot T_{ns/dq}^{-1} \cdot i_{dq} - \frac{dT_{ns/dq}}{dt} \cdot \lambda_{ns} \cdot \psi_f$$

The accuracy of the parameters will affect the dynamic response of the motor control system

# Sensorless Control Based on Model Reference Adaptive System

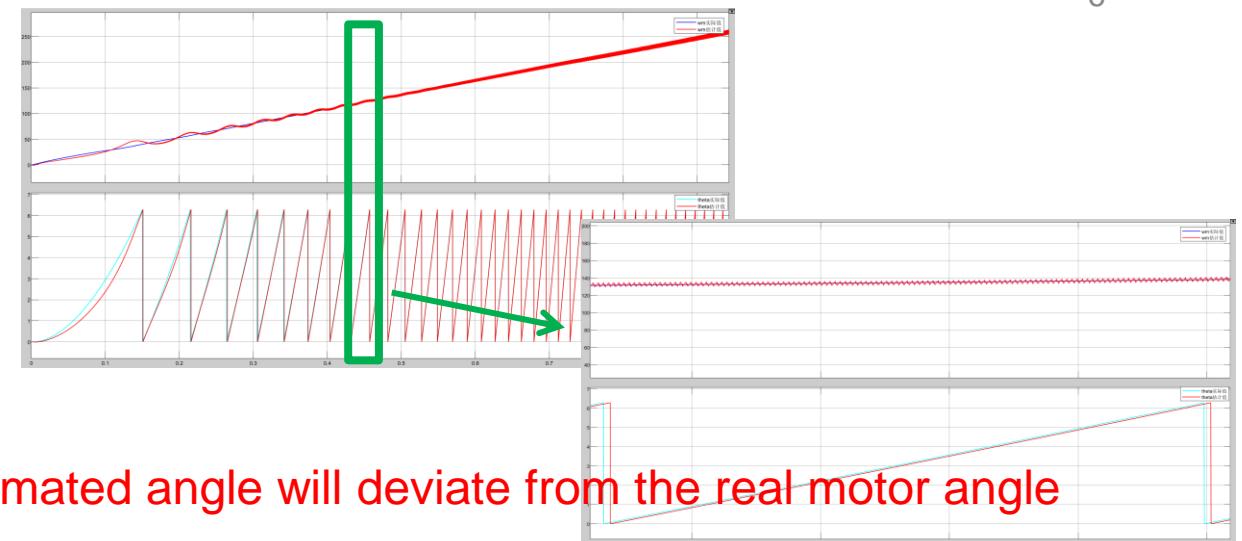
The adjustable model is obtained according to the stator voltage equation of the motor.



Model reference adaptive system(MRAS) is widely used in sensorless motor control. MRAS consists of three main components: **reference model**, **adjustable model**, and **adaptive law**.

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\frac{r_s}{L_d} & \omega_e \frac{L_q}{L_d} \\ -\omega_e \frac{L_d}{L_q} & -\frac{r_s}{L_q} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{u_d}{L_d} \\ \frac{u_q}{L_q} \end{bmatrix}$$

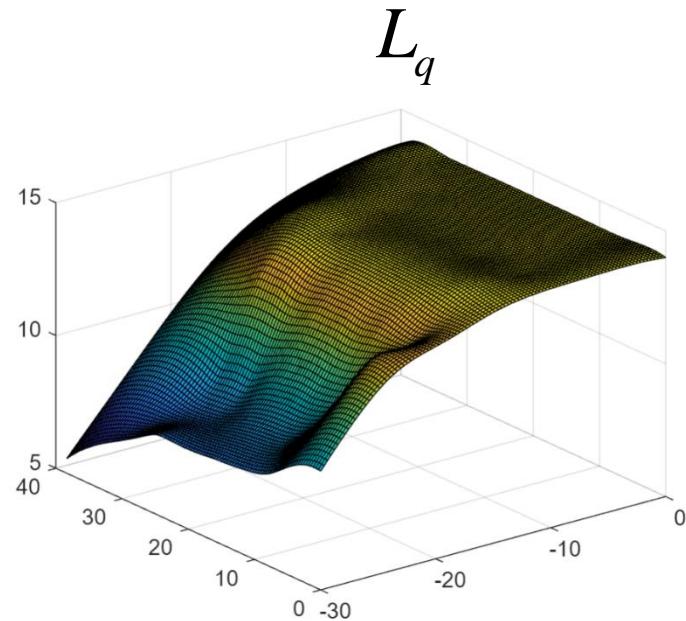
When a parameter in an adjustable model deviates from real model:



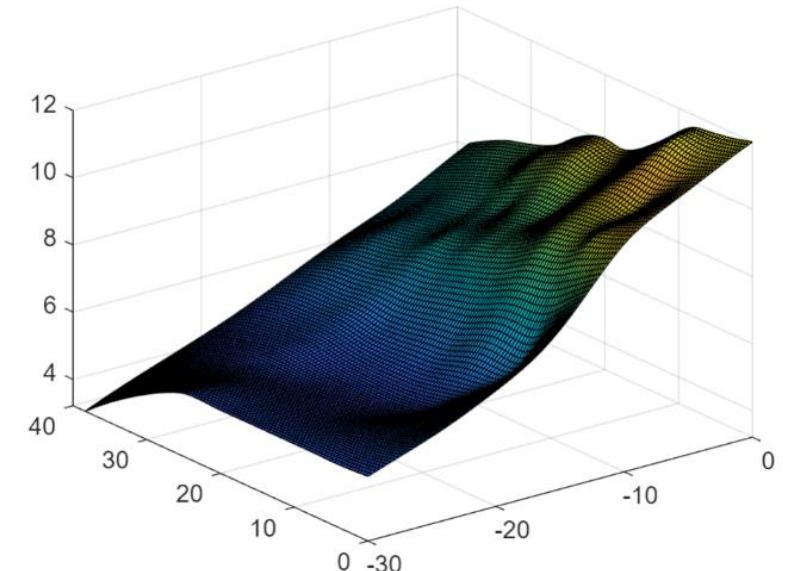
The estimated angle will deviate from the real motor angle

According to the above research on vector control and speed sensorless algorithm, it can be seen that the accurate identification of parameters is the basis of high-performance algorithm research

Based on the actual calibration results of the inductance parameters in the debugging process of the positionless algorithm:



$L_d$



Compared to resistance parameters, the measurement of motor inductance and magnetic flux parameters is more challenging, and their values are more significantly influenced by motor operation.

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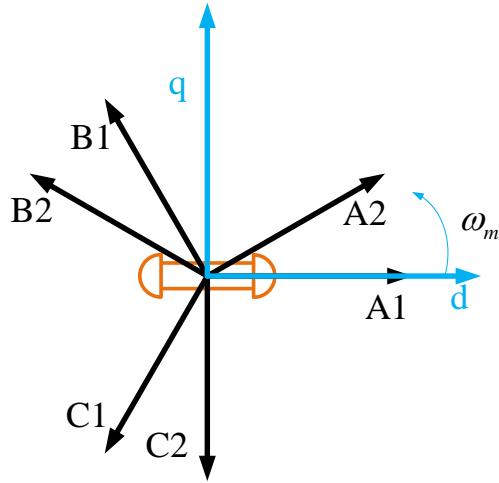
Base on Recursive Least Square Method

4. Simulation & Results

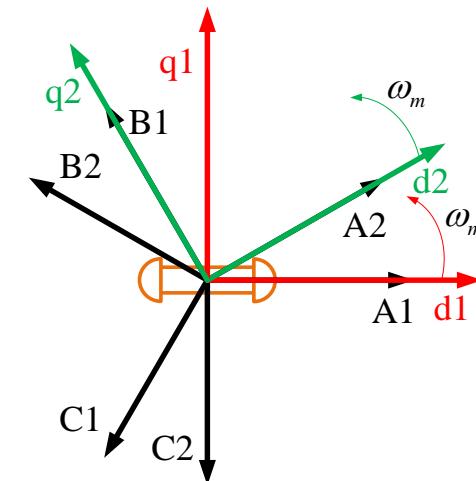
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# Mathematical Model of DTP-PMSM

DTP-PMSM according to the coordinate projection relationship can be divided into: 1. based on Vector Space Decomposition(VSD) coordinate transformation, 2. based on dual-dq coordinate transformation



⋮



$$\begin{bmatrix} u_d \\ u_q \\ u_x \\ u_y \end{bmatrix} = R_s \begin{bmatrix} i_d \\ i_q \\ i_x \\ i_y \end{bmatrix} + \begin{bmatrix} L_d & 0 & 0 & 0 \\ 0 & L_q & 0 & 0 \\ 0 & 0 & L_{aal} & 0 \\ 0 & 0 & 0 & L_{aal} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \\ i_x \\ i_y \end{bmatrix} + \omega_e \psi_m \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \omega_e \psi_m \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_{d1} \\ u_{q1} \\ u_{d2} \\ u_{q2} \end{bmatrix} = R_s \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} + \begin{bmatrix} L_d & 0 & M_d & 0 \\ 0 & L_q & 0 & M_q \\ M_d & 0 & L_d & 0 \\ 0 & M_q & 0 & L_q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} + \omega_e \begin{bmatrix} 0 & -L_q & 0 & -M_q \\ L_d & 0 & M_d & 0 \\ 0 & -M_q & 0 & -L_q \\ M_d & 0 & L_d & 0 \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} + \omega_e \psi_m \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Which includes unknown parameters: d-axis inductances  $L_d$  & q-axis inductances  $L_q$  & leakage inductances  $L_{aal}$  & mutual inductances  $M_d / M_q$

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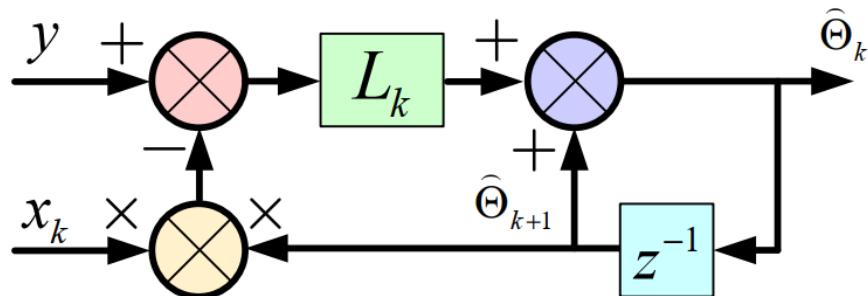
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# The Recursive Least Square Method

The basic principle of recursive least square method is to seek the best parameter value of the matrix to be estimated by recursing constantly according to the actual measurement data.



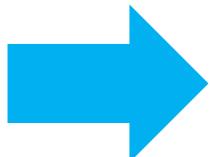
$$\begin{cases} P_k = \frac{1}{\lambda} \left( P_{k-1} - \frac{P_{k-1} x_k x_k^T P_{k-1}}{\lambda + x_k^T P_{k-1} x_k} \right) \\ L_k = \frac{P_{k-1} x_k^T}{\lambda + x_k^T P_{k-1} x_k} \\ \hat{\Theta}_k = \hat{\Theta}_{k+1} + L_k (y - x_k \hat{\Theta}_{k+1}) \end{cases}$$

The algorithm can be realized by adjusting the parameter values of the forgetting factor to have faster convergence speed and stability of the recognition results.

# DTP PMSM Parameter Estimation strategy

By deducing the stator equation of motor, a suitable recursive matrix for parameter identification by least square method is constructed.

$$\begin{bmatrix} \frac{i_d}{dt} \\ \frac{i_q}{dt} \\ \frac{i_x}{dt} \\ \frac{i_y}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1}{L_d} & \frac{1}{L_d} & \frac{L_q}{L_d} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L_q} & \frac{1}{L_q} & \frac{L_d}{L_q} & \frac{\psi_f}{L_q} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{L_{aal}} & \frac{1}{L_{aal}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{L_{aal}} & \frac{1}{L_{aal}} & 0 \end{bmatrix} \begin{bmatrix} u_d \\ -R_s i_d \\ \omega_e i_q \\ u_q \\ -R_s i_q \\ -\omega_e i_d \\ -\omega_e \\ u_x \\ -R_s i_x \\ u_y \\ -R_s i_y \end{bmatrix}$$



$$\left\{ \begin{array}{l} \hat{\Theta}_k = \begin{bmatrix} \frac{1}{L_d} & \frac{1}{L_d} & \frac{L_q}{L_d} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L_q} & \frac{1}{L_q} & \frac{L_d}{L_q} & \frac{\psi_f}{L_q} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{L_{aal}} & \frac{1}{L_{aal}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{L_{aal}} & \frac{1}{L_{aal}} \end{bmatrix} \\ y(k) = \begin{bmatrix} \frac{di_d}{dt} & \frac{di_q}{dt} & \frac{di_x}{dt} & \frac{di_y}{dt} \end{bmatrix}^T \\ x(k) = \begin{bmatrix} u_d & -R_s i_d & \omega_e i_q & u_q & -R_s i_q & -\omega_e i_d \\ -\omega_e & u_x & -R_s i_x & u_y & -R_s i_y \end{bmatrix}^T \end{array} \right.$$

The dq axis of the stator voltage equation based on VSD coordinate transformation is decoupled from each other, so its recursive equation is relatively simple to construct, and the matrix to be identified is obtained by direct transformation:

# DTP PMSM Parameter Estimation strategy

The stator voltage equation based on double dq coordinate transformation is relatively complicated to identify its parameters because its dq axis is coupled with each other. Firstly, the following recursive equation form is obtained by adding/subtracting the parameters to be identified.

$$\left\{ \begin{array}{l} L_d \frac{di_{d1}}{dt} + M_d \frac{di_{d2}}{dt} = u_{d1} - R_s i_{d1} + \omega_e L_q i_{q1} + \omega_e M_q i_{q2} \\ \hat{\Theta}_{dq} = \begin{bmatrix} \alpha_1 & \alpha_1 & 0 & 0 & 0 & 0 & \alpha_5 & \alpha_5 & 0 \\ 0 & 0 & \alpha_2 & 0 & \alpha_2 & \alpha_2 & \alpha_6 & \alpha_6 & 0 \\ 0 & \alpha_3 & 0 & -R_s i_{q1} & \alpha_3 & 0 & \alpha_7 & 0 & \alpha_9 \\ M_q \frac{di_{q1}}{dt} + M_d \frac{di_{q2}}{dt} = u_{q1} - R_s i_{q1} - \omega_e L_d i_{d1} - \omega_e M_d i_{d2} - \omega_e \psi_f \\ M_d \frac{di_{d1}}{dt} + L_d \frac{di_{d2}}{dt} = u_{d2} - R_s i_{d2} + \omega_e M_q i_{q1} + \omega_e L_q i_{q2} \\ M_q \frac{di_{q1}}{dt} + L_q \frac{di_{q2}}{dt} = d\dot{t}_{q2} - R_s \dot{i}_{q2} - \omega_e M_d i_{d1} \frac{di_{d1}}{dt} - \omega_e L_d \dot{i}_{d2} - \omega_e \dot{\psi}_f \\ y(k) = \begin{bmatrix} \frac{di_{d1}}{dt} + \frac{di_{d2}}{dt} & \frac{di_{q1}}{dt} + \frac{di_{q2}}{dt} & \frac{di_{d1}}{dt} - \frac{di_{d2}}{dt} & \frac{di_{q1}}{dt} - \frac{di_{q2}}{dt} \end{bmatrix}^T \\ x(k) = \begin{bmatrix} u_{d1} & u_{d2} & u_{q1} & u_{q2} & -R_s i_{d1} & -R_s i_{d2} & -R_s i_{q1} & -R_s i_{q2} \end{bmatrix} \end{array} \right.$$

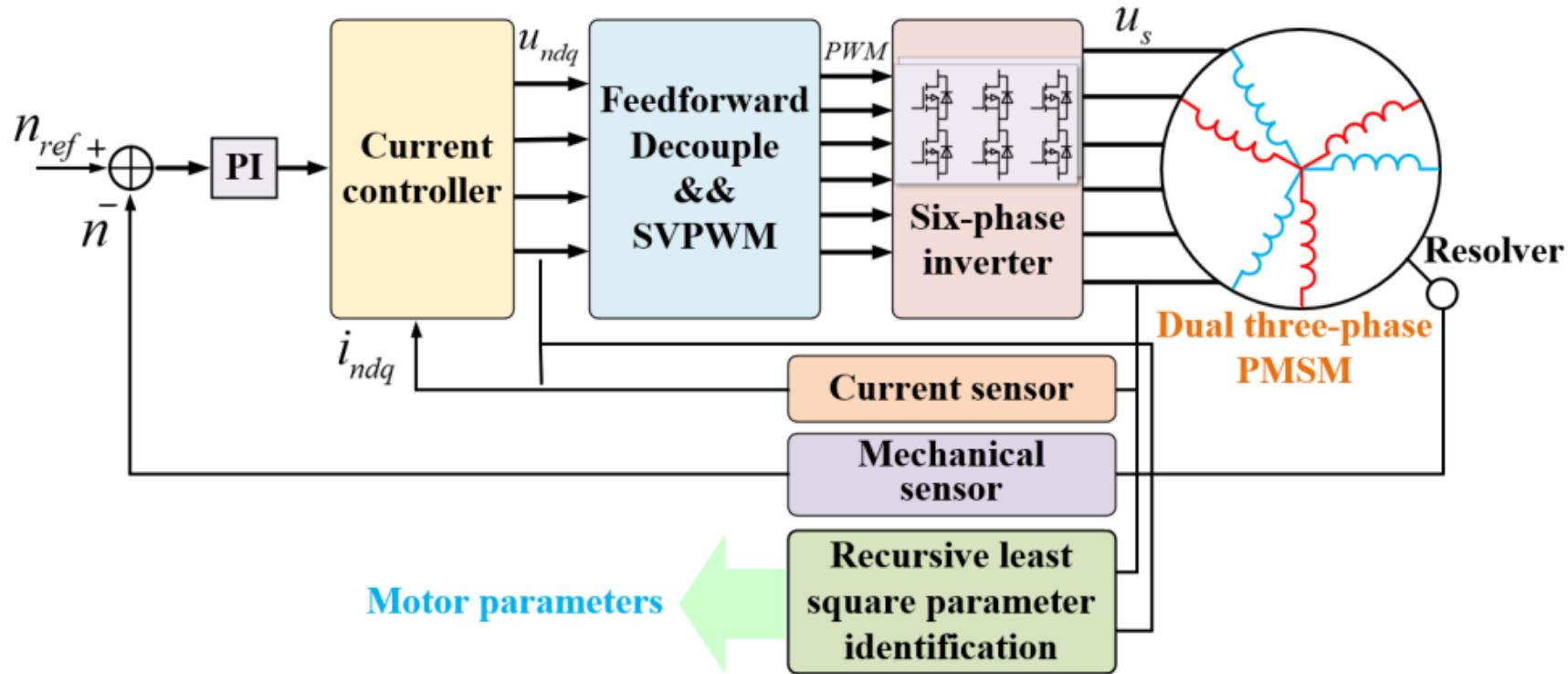
$$\begin{aligned} \alpha_1 &= \frac{1}{(L_d + M_d)}; \alpha_2 = \frac{1}{(L_q + M_q)}; \alpha_3 = \frac{1}{(L_d - M_d)}; \\ \alpha_4 &= \frac{1}{(L_q - M_q)}; \alpha_5 = \frac{(L_q + M_q)}{(L_d + M_d)}; \alpha_6 = \frac{(L_d + M_d)}{(L_q + M_q)}; \\ \alpha_7 &= \frac{(L_q - M_q)}{(L_d - M_d)}; \alpha_8 = \frac{(L_d - M_d)}{(L_q - M_q)}; \alpha_9 = \frac{2\psi_f}{(L_q + M_q)}; \end{aligned}$$

- Increasing the rank of the estimation matrix by adding input voltage and current state values, such as (harmonic) current injection ( $i_{d1} - i_{d2}$ ) +  $\omega_e (L_q - M_q) i_{q1} + \omega_e (-L_q + M_q) i_{q2}$ )

- Employing segmented estimation, reducing the number of parameters to be identified within each estimation cycle to enhance the estimation accuracy

# DTP PMSM Parameter Estimation strategy

The parameter estimation strategy for DTP PMSM system based on the recursive least squares method



When the DTP PMSM system is in operation, taking into account the noise and error in the sampling process, the steady state voltage and current state value of the motor will fluctuate within a certain range, while the average value remains constant, so this paper in the state of the amount of computing using the design of low pass filter to filter out the interference brought about by the high-frequency noise.

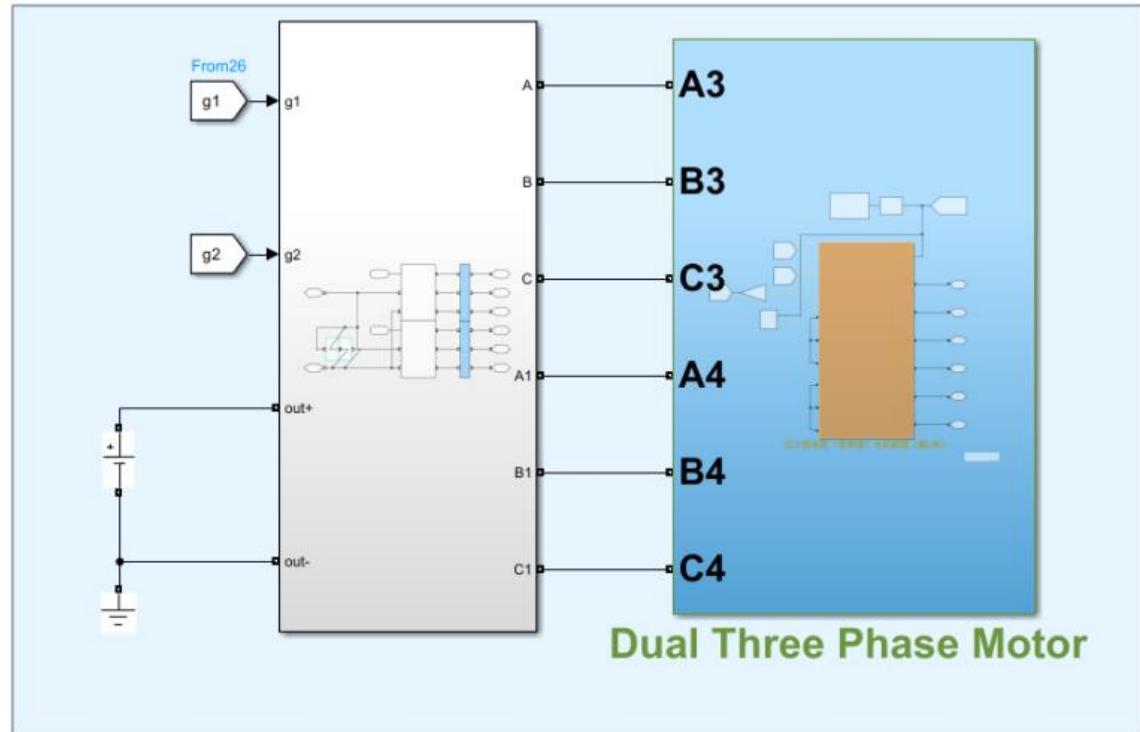
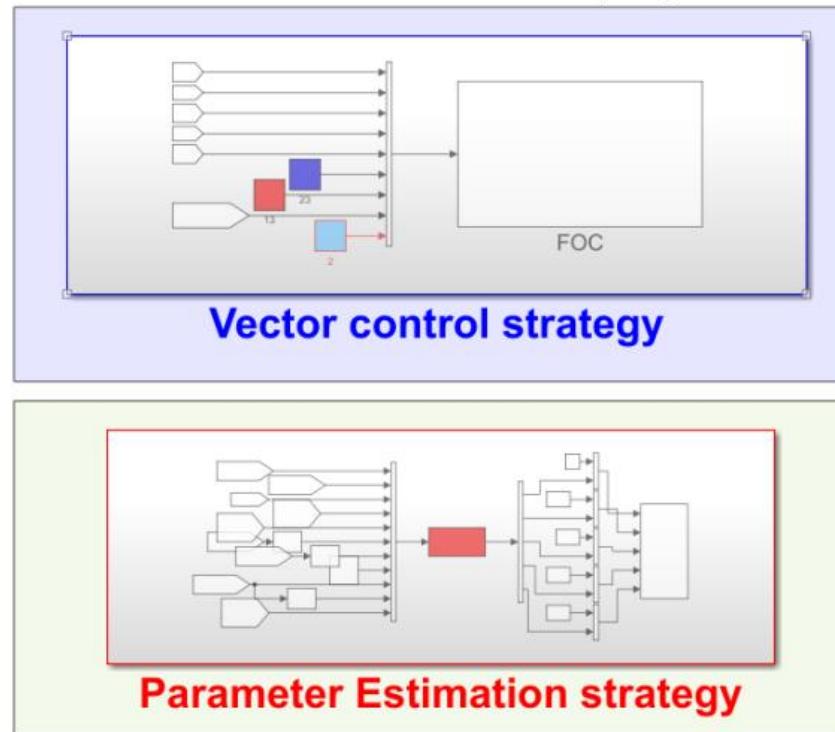
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# Simulation results

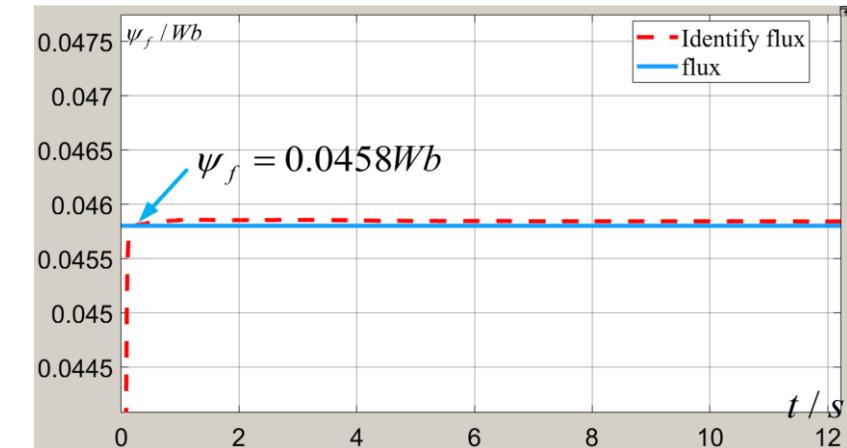
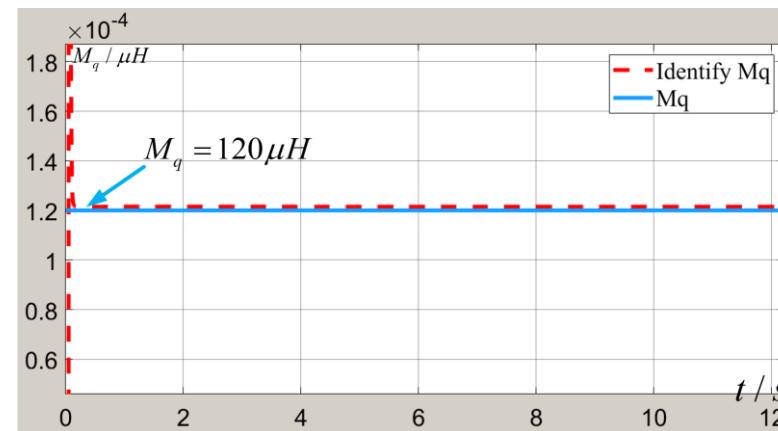
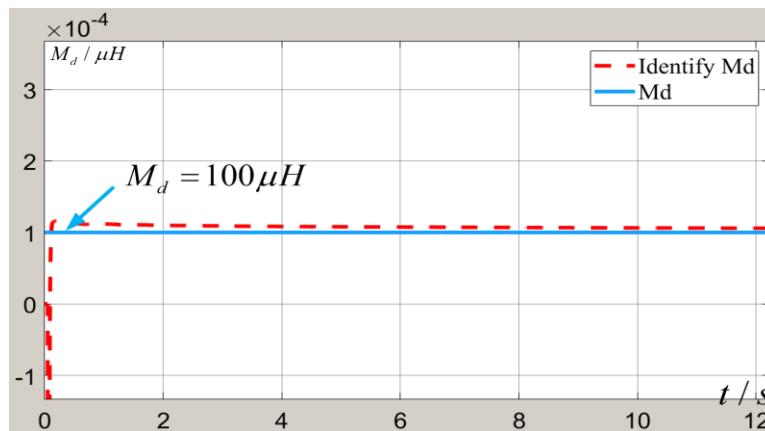
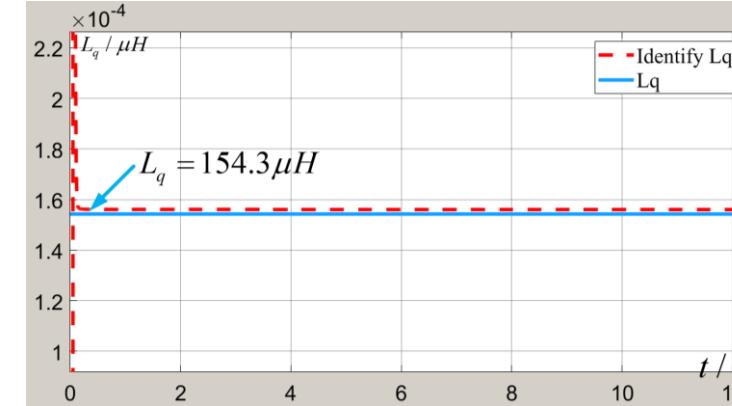
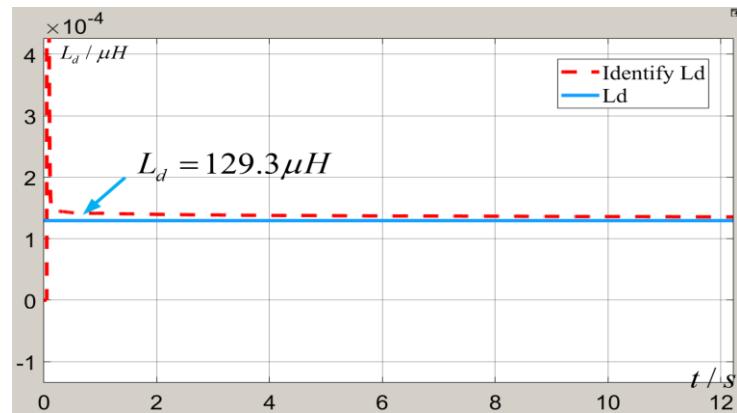
In order to verify the correctness of the proposed DTP PMSM parameter estimation strategy, the control simulation model of the electric propulsion system, which consists of vector control, parameter estimation strategy and DTP PMSM system, is established in SIMULINK.

Parameter	Value
d-axis inductance	129.3 $\mu$ H
q-axis inductance	154.3 $\mu$ H
d-axis mutual inductance	100 $\mu$ H
q-axis mutual inductance	120 $\mu$ H
leakage inductance	5 $\mu$ H
Permanent magnet flux	0.0458Wb
Stator resistance	34.68m $\Omega$
Number of pole pairs	20



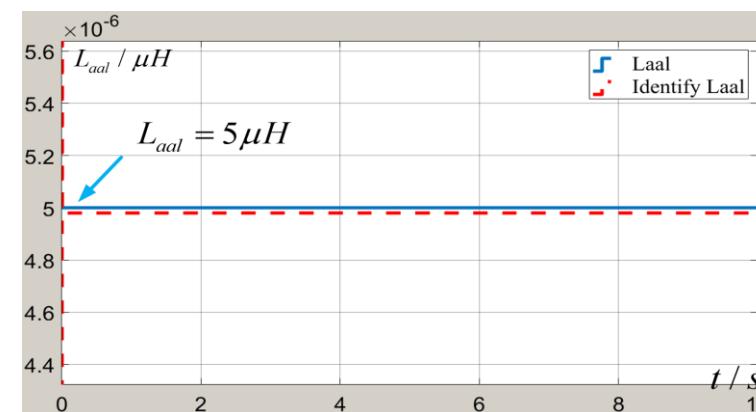
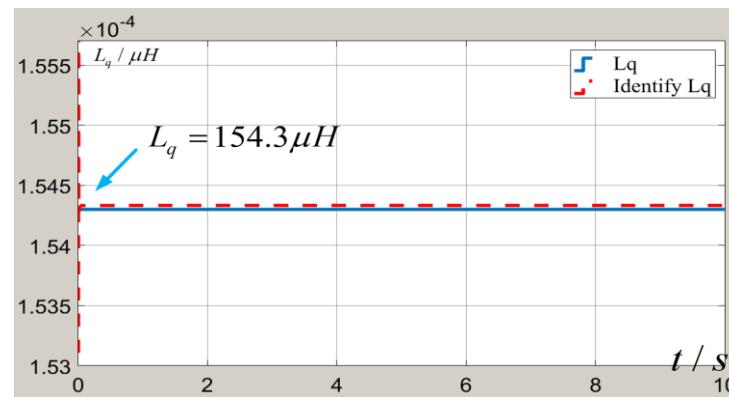
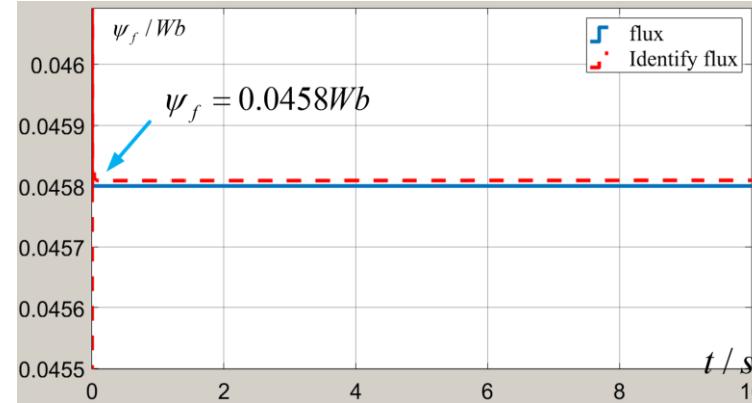
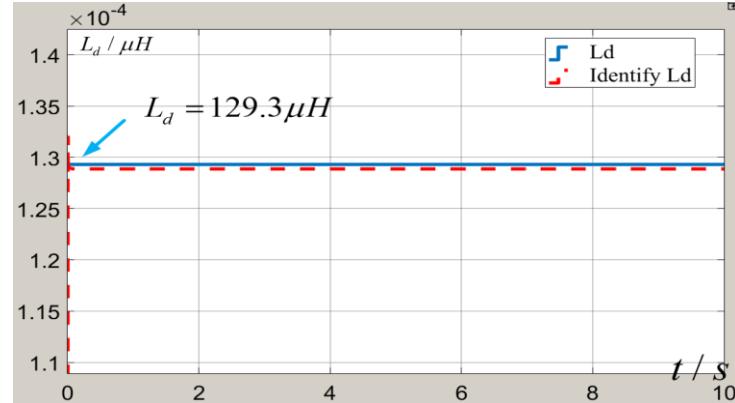
# Parameter estimation based on dual-dq coordinate

— Real parameter of motor — — Parameter estimation result ↵



# Parameter estimation based on VSD coordinate

Real parameter of motor      Parameter estimation result



According to the simulation results, the proposed parameter estimation strategy is able to realize accurate estimation of real parameters based on both control models and has a fast convergence speed. The feasibility and accuracy of the proposed parameter estimation strategy for DTP PMSM based on the iterative least squares method are demonstrated.

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# Conclusion

- ◆ The necessity of applying parameter estimation algorithm to DTP PMSM is analyzed.
- ◆ Proposing a parameter estimation strategy for inductance and magnetic flux linkage based on the least squares method for DTP PMSM control systems.
- ◆ According to the dual dq model and VSD model, the parameter recurrence matrix is derived respectively, and the DTP PMSM parameter estimation method based on two coordinate transformation methods is introduced respectively.

DTP PMSM have significant advantages in high reliability requirements and large capacity operation, and the research on parameter estimation strategy in this paper can help to provide effective data support for the realization of high performance control, such as sensorless control, which is expected to be used in the future electrified transportation electric propulsion system.

# Further research

- ※ Experimental verification of parameter estimation of DTP PMSM method.
- ※ The proposed parameter identification method is applied to the sensorless algorithm

# Thanks For Listening